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Research Note

The Role of the Gouy Phase Anomaly in the Unification of the Geometric and Physical Models for the Propagation of Focussed Fields

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Abstract

Although first discovered over a century ago by the scientist of the same name, the Gouy phase effect continues to awaken considerable interest in the scietific community. In this work the authors study the spherical wave (as one of the most illustrative tools in optics) and the coexistence therein of geometric and diffractive behaviours. The Gouy phase anomaly plays a prominent role in this analysis.

Keywords: · diffraction · geometrical optics · Gouy phase ·

1 Introduction

From the very first moment when optics is encountered in the early stages of one's education it is presented as divided into two, almost independent, parts: its geometrical and physical variants. It is easy to overlook the fact that both of them describe the same physical entity—light—and thus, rather than separate blocks, the two simply correspond to different levels of approximation.

It then naturally follows that the more drastic approximation of the two (namely, geometrical optics) must somehow be fully contained in the more rigorous approach (physical optics). It also means that the frontier between the two, rather than a sharp border, would be characterised by a gradual decrease in accuracy of the more strongly approximated model.

There is one example particularly well suited to illustrate the above: the spherical wave, a fundamental concept in both branches of optics. It is generally accepted that geometrical optics can accurately describe a spherical wave, except for the area around and including the focus. This assertion, however, is seldom supported by any meaningful quantification. One of the reasons therefor is, evidently, the sheer impossibility to compute the error of the geometric model with respect to the diffractive one as long as one deals in rays and the other in vector fields.

The answer is then to take the tenets of geometrical optics and apply them to Maxwell's equations, so that a new set of differential equations is obtained. The solution to these new equations will again be a vector field, but its behaviour will be dictated by the geometric approximation [1]. We shall refer to these new equations as the "geometric field equations", a solver for which was implemented by our team [2], thus enabling, in practice, the comparison of the two models, in the manner which is described in Section 2.

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2 Results of the simulation experiment

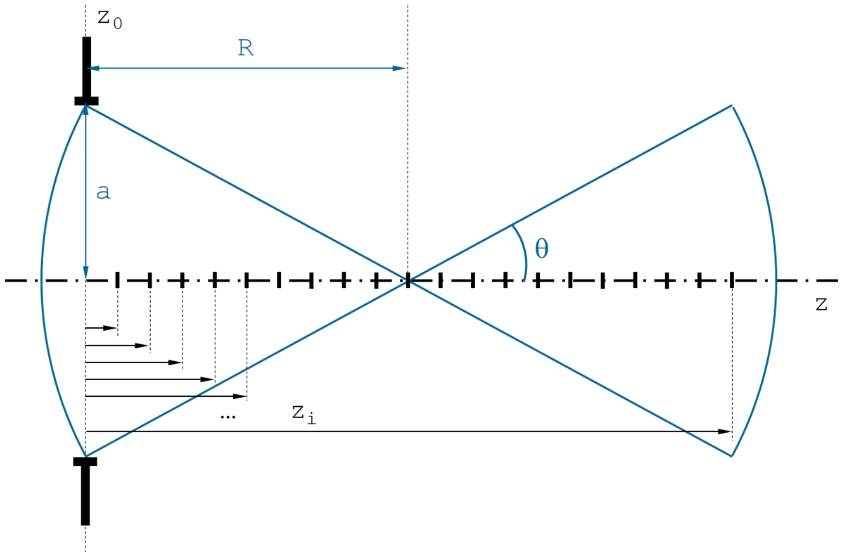

Figure 1: The figure aims to illustrate the procedure followed in the simulation experiments which constitute the cornerstone of this work.

The object of this work is to quantify the error incurred by the following operation: propagating, according to the geometric field equations, an ideal convergent spherical wave from a position z_0 (assumed to be far enough away from the focus) towards the focus, to a position denoted by z_1. The spherical wave is truncated at z_0 by an aperture of radius a.

In order to compute this error the same operation is carried out, but this time employing the standard Maxwell equations, or, in other words, via the spectrum-of-plane-waves (SPW) propagation operator, to obtain a rigorous result of the propagated field at z_1. The relative standard deviation σ of the geometrically propagated field, $\boldsymbol{E}_{\text{geom}}\left(x, y; z_i\right)$, and the field propagated with SPW, $\boldsymbol{E}_{\text{diff}}\left(x, y; z_i\right)$,

$$\sigma\left[\boldsymbol{E}_{\text{geom}}\left(x, y; z_i\right), \boldsymbol{E}_{\text{diff}}\left(x, y; z_i\right)\right] = \frac{\int_{-\infty}^{\infty} \left|\left|\boldsymbol{E}_{\text{geom}}\left(x, y; z_i\right) - \boldsymbol{E}_{\text{diff}}\left(x, y; z_i\right)\right|\right|^2 \mathrm{d}x\mathrm{d}y}{\int_{-\infty}^{\infty} \left|\left|\boldsymbol{E}_{\text{diff}}\left(x, y; z_i\right)\right|\right|^2 \mathrm{d}x\mathrm{d}y} \tag{1}$$

is then calculated using the SPW result as reference. If this is done not only for $z_0 \to z_1$, but also for $z_0 \to z_2, \ldots, z_0 \to z_i$, eventually also crossing the focus, the error of the geometric model with respect to the rigorous SPW can be plotted against the distance from the focus, and thus the object of the analysis is realised. A schematic representation of the procedure is shown in Fig. 1.

One such plot of the deviation is shown in Fig. 2. The blue curve corresponds to the relative standard deviation of the geometrically propagated field with respect to the same field propagates with SPW (Eq. 1. For the red curve, the calculation is similar, only this time $\boldsymbol{E}_{\text{geom}}\left(x, y; z_i\right)$ is multiplied by an arbitrary complex constant (per x, y plane) $\gamma\left(z_i\right)$ which minimises the value of σ for that particular plane.

3 Analysis and conclusions

Let us draw some conclusions from Fig. 2. We shall first focus on the blue curve. Before the spherical wave reaches its focus, the results are pretty much what one would expect: the deviation remains at

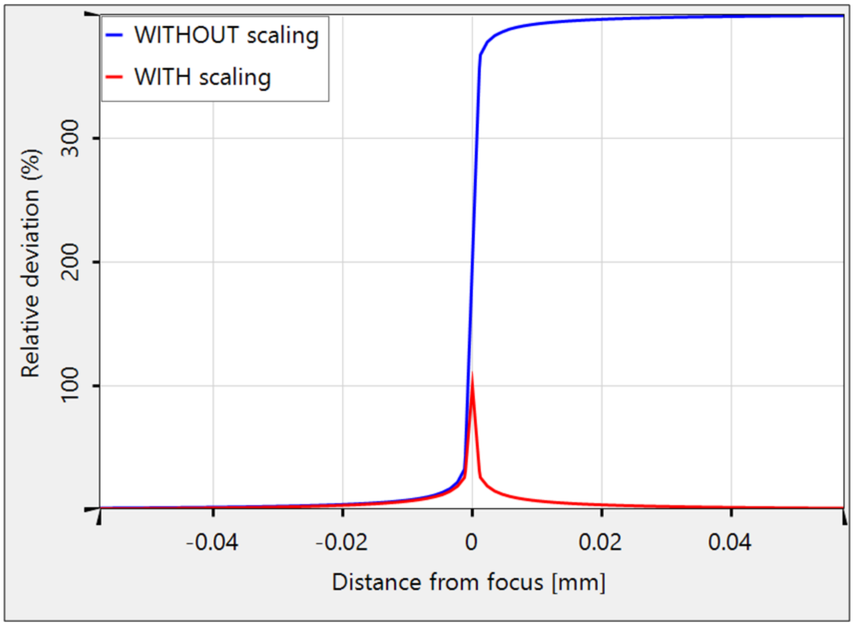

Figure 2: The figure shows a plot of the relative standard deviation (Eq. 1) of the geometrically propagated field with respect to the same field propagated with SPW. For the blue curve the geometric and the diffractive results were compared as-is, for the red curve the geometric result was multiplied by a constant $\gamma(z_i) \in \mathbb{C}$ which minimises the value of σ.

a very low level (geometrical optics works correctly) until diffraction effects take over and the geometrical field equations are no longer able to accurately predict the field; consequently the deviation shoots up.

But the more thought-provoking result comes when we cross the focus. Propagating the field from the geometric zone[1] on one side of the focus, through the focus, to the geometric zone on the other side using geometric propagating throughout is a technique which, albeit widespread, works on a blatantly false assumption: namely, that the field behaves geometrically throughout the entire propagation step. The blue curve, for which the deviation never comes down once the field has traversed the focus, seems to point to the incorrectness not only of the assumption, but also of the result.

But what if, as in the case of th red curve, a complex constant per x, y plane $\gamma(z_i)$ was allowed to be multiplied on the geometric result, on condition that it minimise the deviation of the geometric result with respect to the diffractive one? Let us emphasise that such an operation does not modify the overall "shape" of the field in the plane in question. It can just scale it or superimpose a constant phase term on it. Should the shape of the geometric result differ too strongly from the rigorous diffractive one, the best γ can do is multiply the geometric result by such a low value that the deviation does not exceed $100\,\%$.

And this is indeed what happens at the focus for the red curve: within the diffractive zone, the geometric field equations are unable to yield a correct result, and thus σ increases smoothly to peak at this position with a value of $100\,\%$. But behind the focus, the red curve decreases again. **This implies that for the geometric zone on the other side of the focus, the geometric field equations do predict the field correctly, except for a constant factor.**

Closer inspection of $\gamma(z_i)$ reveals that, on both sides of the focus, it has unit amplitude and is thus a pure phase term. Moreover, this term represents a phase difference of π between both sides of the focus:

[1] We shall use the term "geometric zone" to refer to those regions of space in which the geometric Fourier transform [3] gives an accurate description. When this condition does not hold we speak of a "diffractive zone".

what γ represents, in short, is the well-known Gouy phase shift [4], the minus sign necessary to change a focussed field from convergent to divergent, $-\exp\left(ß\,|\boldsymbol{k}|\,|\boldsymbol{r}|\right)/\,|\boldsymbol{r}| \to \exp\left(ß\,|\boldsymbol{k}|\,|\boldsymbol{r}|\right)/\,|\boldsymbol{r}|$, which, as per the theory of complex numbers, can also be understood as a π phase term [1].

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