Simulation Example

Second harmonic generation under phase-mismatched conditions

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Abstract
Second harmonic generation is an important effect of nonlinear optics. It is well discussed for cw processes, so in this project we want to show its properties for ultrashort pulses through the use of the simulation software VirtualLab Fusion [1]. The fundamental principles are explained in my previous report 'Second harmonic generation under phase-matching conditions'. In this report we discuss the physical effects when phase-matching conditions are not met.

Keywords: Second harmonic generation, ultrashort pulses

Physics

The last report showed the fundamental equations that describe the second harmonic generation process can be written as [2]:

$$\partial_z \hat{E}_F(\omega_1, z) = -\alpha_1 \int d\omega_2 \hat{E}_F^*(\omega_2, z)\hat{E}_{SH}(\omega_2 + \omega_1, z)e^{+i\Delta k(\omega_1,\omega_2+\omega_1)z} \tag{1}$$

$$\partial_z \hat{E}_{SH}(\omega_3, z) = -\alpha_3 \int d\omega_1 \hat{E}_F(\omega_1, z)\hat{E}_F(\omega_3 - \omega_1, z)e^{+i\Delta k(\omega_1,\omega_3-\omega_1)z} \tag{2}$$

with

$$\hat{E}_i(\omega_i, z) = E_i(\omega_i, z)e^{ik_i L} \tag{3}$$

$$\Delta k(\omega_1, \omega_2) = (\omega_1 + \omega_2)n(\omega_1 + \omega_2) - \omega_2 \cdot n(\omega_2) - \omega_1 \cdot n(\omega_1) \tag{4}$$

$$\alpha_i = \frac{\omega_i d_{eff}}{c_0 n_i} \tag{5}$$

which is taking care of the linear dispersion.

Here, d_{eff} represents the effective nonlinearity, E_F and E_{SH} the fundamental field and its corresponding second harmonic and L the length of the crystal. $c_0 = \frac{1}{\epsilon_0 \mu_0}$ is the vacuum speed of light and n the refractive index at various frequencies.

The important factor for the efficiency of the energy transfer from the fundamental pulse to the second harmonic is Δk. Contrary to the last report, we now want to focus on second harmonic generation when Δk is not minimized. This means, that the intensity of the second harmonic will always be much lower then the intensity of the fundamental pulse. Therefore, we can always assume that the fundamental pulse is not depleted. We then set $\partial \hat{E}_F = 0$ and easily solve the system of equations to

$$E_{SH}(\omega_3) = -\alpha_2 \int d\omega_1 E_F(\omega_1)E_F(\omega_3 - \omega_1)[\frac{e^{+i\Delta k(\omega_1,\omega_3-\omega_1)L} - 1}{\Delta k}] \tag{6}$$

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with

$$\Delta k(\omega_1, \omega_2) = (\omega_1 + \omega_2) n(\omega_1 + \omega_2) - \omega_2 \cdot n(\omega_2) - \omega_1 \cdot n(\omega_1) \tag{7}$$

$$\alpha_i = \frac{\omega_i d_{eff}}{c_0 n_i}. \tag{8}$$

. [3]

Frequency filtering in frequency domain

If the phase-matching conditions are not met, the original field and its generated second harmonic pulses do no longer travel with the same speed. This results in destructive interference. The occurrence of the destructive interference is depended on the frequency of the original field and the length of the crystal. By looking at a certain coherence length L_{coh} we can see for what frequencies of the original field this destructive interference destroys the second harmonic. The coherence length is defined as

$$L_{coh} = \frac{2\pi}{\Delta k}. \tag{9}$$

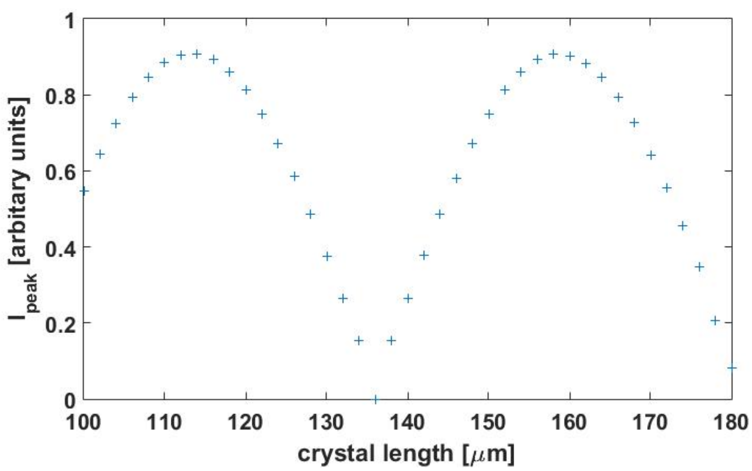

Figure 1: Peak Amplitude of a monochromatic 800 nm light shown over different crystal lengths. The coherence length is about 40 $\mu\mathrm{m}$.

It is expected that the intensity profile of an second harmonic that originates from a monochromatic field goes like a squared sinus. The frequency of this squared sinus is depended on the frequency of the monochromatic wave. Because every pulse consists of many frequencies, for every crystal length L we can find a bunch of frequencies, where destructive interference happens. For longer crystal lengths L this means, that we will observe an filtering of the second harmonic in frequency domain. All frequencies are filtered out, when the crystal length is an exact multitude of their corresponding coherence length.

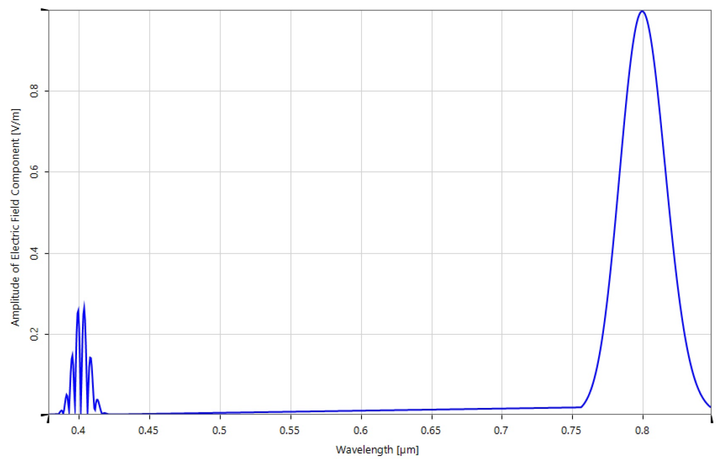

Figure 2: Fundamental pulse and second harmonic for BBO after 700 $\mu\mathrm{m}$. In this simulation we assumed an incident angle of $26°$. Even small derivations from the phase-matching angle lead to an enormous decrease of the amplitude of the second harmonic in comparision to the fundamental pulse.

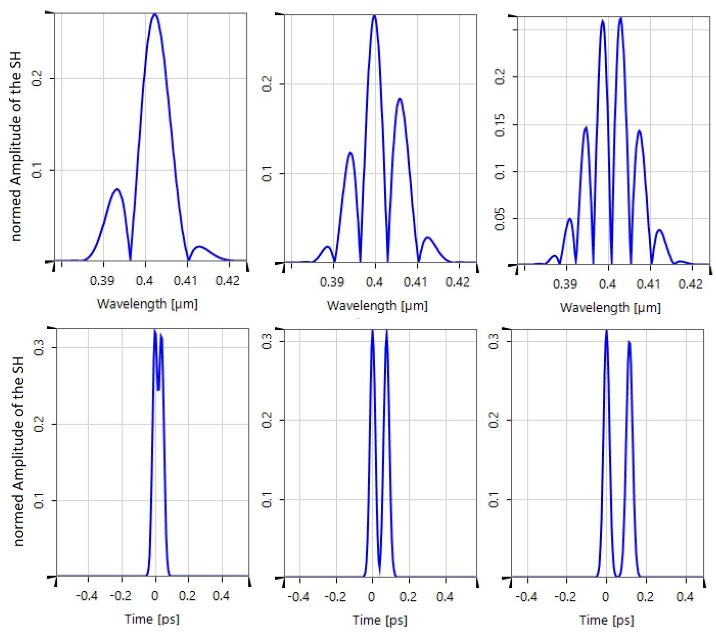

Figure 3: Second harmonic at various crystal lengths in the non-phase-matched case. The angle is 26 and the nonlinear material BBO. The destructive interference is resulting in an filtering of certain frequencies depending on the length of the crystal.

Pulse splitting in time domain

The fourier transformation of such a profile will give multiple peaks in time domain. Physically, the pulse splitting is explained as in the second harmonic generation process are actual two different pulses created due to the cascending nonlinearity of the crystal [4]. This effect only is observable for ultrashort pulses. The time delay of these two pulses depends on the crystal length L and is generally in the order of picoseconds. So it is quite natural that this effect will disappear when the temporal dilation exceeds this values.

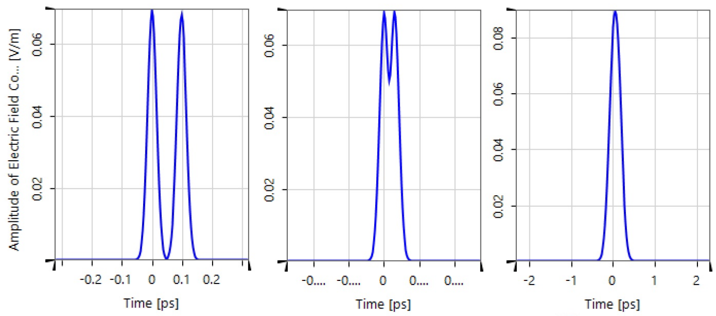

Figure 4: Second Harmonic in time domain with pulse lengths of 35 fs, 100 fs and 250 fs. The splitting effect disappears at higher lengths as it is overlayed by the temporal broadness of the second harmonic.

Bibliography

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[4] J-H Lin and K-H Lin. Multiple pulsing and harmonic mode-locking in an all-normal-dispersion nd:gdvo 4 laser using a nonlinear mirror. *Journal of Physics B: Atomic, Molecular and Optical Physics*, 43(6):065402, 2010.