**Research Note**

# Geometric field tracing through GRIN media and fibers (II)

Huiying Zhong*, Site Zhang, Rui Shi, Christian Hellmann, Frank Wyrowski

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*Correspondence: huiying.zhong@uni-jena.de

**Abstract**
The damping of the media, which is represented by the imaginary part of complex refractive index, affects the electromagnetic field when it travels along the ray path. In previous research, the effect is not explicitly discussed or even neglected. In this report, we propose the mathematical solusion of equation of ray path and normlized field in GRIN media with damping.

**Keywords:** ray tracing; graded-index media(GRIN); geometric field tracing; damping media

## Introduction

In the concept of geometric field tracing, the smart ray, which contains field information, is traced. [1] The smart ray tracing in GRIN is done by solving the following two differential equations.

$Equation\ of\ Ray\ Path$:

$$\frac{d}{ds}[\tilde{n}(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}] = \boldsymbol{\nabla}\tilde{n}(\boldsymbol{r}) \tag{1}$$

where $\tilde{n}(\boldsymbol{r})$ is the complex refractive index, which denotes that media is damping and $s$ denotes the arc length of the ray.

$Equation\ of\ Normalized\ Field$ :

$$\tilde{n}(\boldsymbol{r})\frac{d\tilde{\boldsymbol{u}}(\boldsymbol{r})}{ds} = -[\tilde{\boldsymbol{u}}(\boldsymbol{r}) \cdot \frac{\boldsymbol{\nabla}\tilde{n}(\boldsymbol{r})}{\tilde{n}(\boldsymbol{r})}]\tilde{n}(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds} \tag{2}$$

where $\tilde{\boldsymbol{u}}(\boldsymbol{r})$ denotes the complex three dimensional vector of normalized electric field at position $\boldsymbol{r}$.

Compared with the two equations in [2], the equations in this report consider complex refractive index. Vectorial geometric field tracing is done by solving the two equations numerically and simultaneously. [3]

## Algorithm

The parameters of smart ray, which are calculated (traced) in this algorithm are: (1)position $\boldsymbol{r}$; (2)direction $\hat{\boldsymbol{s}}(\boldsymbol{r})$; (3)arc length $s$; (4)complex

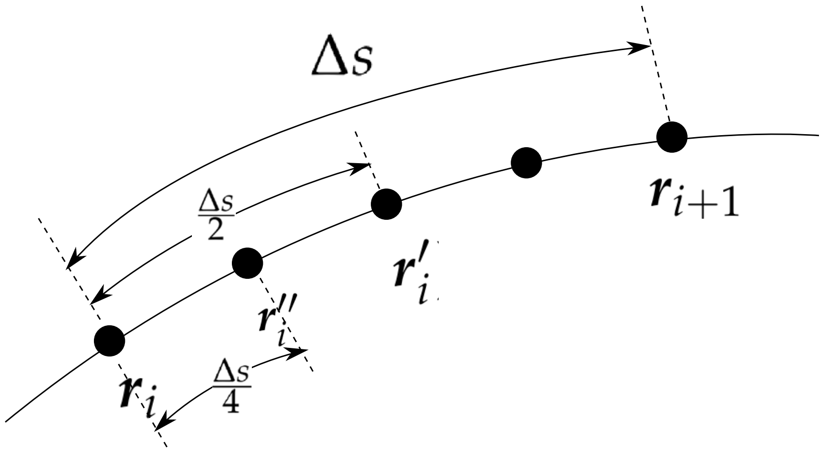


Figure 1: Denotation of parameters in one-step of the iterative calculation.

optical path length OPL$= \int \tilde{n}(\boldsymbol{r})ds$; (5)electric field vector $\tilde{\boldsymbol{E}}(\boldsymbol{r})$ and the normalized electric field vector $\tilde{\boldsymbol{u}}(\boldsymbol{r}) = \tilde{\boldsymbol{E}}(\boldsymbol{r})/||\tilde{\boldsymbol{E}}(\boldsymbol{r})||$.

Eq. (1) and (2) are solved by the iterative method: the fourth order Runge-Kutta methods(RK4). In Fig. 1, some important parameters in the iteration are shown. Compared to [2], the step parameter changes to ray parameter(3) arc length: in the calculation of ray path equation, the step size is $\frac{\Delta s}{2}$, while that in the calculation of normalized field is $\Delta s$.

## Solution of the ray equation

In this part, ray parameters (1) to (4) are calculated. Here we show how to calculate parameters on $\boldsymbol{r}'_i$ from those on $\boldsymbol{r}_i$:

$$\begin{cases} \boldsymbol{r}''_i &= \boldsymbol{r}_i + \frac{\boldsymbol{T}(\boldsymbol{r}_i)}{\tilde{n}(\boldsymbol{r}_i)}\frac{\Delta s}{4} + \frac{\Delta s^2}{32}\left\{\frac{\boldsymbol{D}(\boldsymbol{r}_i)}{\tilde{n}(\boldsymbol{r}_i)} - \frac{\boldsymbol{T}(\boldsymbol{r}_i)[\boldsymbol{D}(\boldsymbol{r}_i)\cdot\boldsymbol{T}(\boldsymbol{r}_i)]}{\tilde{n}^3(\boldsymbol{r}_i)}\right\} \\ \boldsymbol{r}'_i &= \boldsymbol{r}_i + \frac{\boldsymbol{T}(\boldsymbol{r}_i)}{\tilde{n}(\boldsymbol{r}_i)}\frac{\Delta s}{2} + \frac{\Delta s^2}{8}\left\{\frac{\boldsymbol{D}(\boldsymbol{r}_i)}{\tilde{n}(\boldsymbol{r}_i)} - \frac{\boldsymbol{T}(\boldsymbol{r}_i)[\boldsymbol{D}(\boldsymbol{r}_i)\cdot\boldsymbol{T}(\boldsymbol{r}_i)]}{\tilde{n}^3(\boldsymbol{r}_i)}\right\} \\ \boldsymbol{T}(\boldsymbol{r}'_i) &= \boldsymbol{T}(\boldsymbol{r}_i) + \frac{\Delta s}{12}(\boldsymbol{D}(\boldsymbol{r}_i) + 4\boldsymbol{D}(\boldsymbol{r}''_i) + \boldsymbol{D}(\boldsymbol{r}'_i)) \\ \hat{\boldsymbol{s}}(\boldsymbol{r}'_i) &= \frac{\boldsymbol{T}(\boldsymbol{r}'_i)}{n(\boldsymbol{r}'_i)} \\ OPL(\boldsymbol{r}'_i) &= OPL(\boldsymbol{r}_i) + \frac{\Delta s}{8}(\tilde{n}(\boldsymbol{r}_i) + 2\tilde{n}(\boldsymbol{r}''_i) + \tilde{n}(\boldsymbol{r}'_i)) \end{cases} \tag{3}$$

with

$$\begin{cases} \boldsymbol{D}(\boldsymbol{r}) &= \boldsymbol{\nabla}\tilde{n}(\boldsymbol{r}) \\ \boldsymbol{T}(\boldsymbol{r}) &= \tilde{n}(\boldsymbol{r})\hat{\boldsymbol{s}}(\boldsymbol{r}) \end{cases} \tag{4}$$

### Solution of the field equation

In this part ray parameter (5) is calculated. We first calculate the normalized field by solving Eq. (2) Here we show how to calculate normalized field on $\boldsymbol{r}_{i+1}$ from that on $\boldsymbol{r}_i$:

$$\begin{cases} \boldsymbol{k}_1 &= -\{\tilde{\boldsymbol{u}}(\boldsymbol{r}_i) \cdot \dfrac{\boldsymbol{D}(\boldsymbol{r}_i)}{\tilde{n}^2(\boldsymbol{r}_i)}\}\boldsymbol{T}(\boldsymbol{r}_i) \\ \boldsymbol{k}_2 &= -\{[\tilde{\boldsymbol{u}}(\boldsymbol{r}_i) + \frac{\Delta s}{2}\boldsymbol{k}_1] \cdot \dfrac{\boldsymbol{D}(\boldsymbol{r}'_i)}{\tilde{n}^2(\boldsymbol{r}'_i)}\}\boldsymbol{T}(\boldsymbol{r}'_i) \\ \boldsymbol{k}_3 &= -\{[\tilde{\boldsymbol{u}}(\boldsymbol{r}_i) + \frac{\Delta s}{2}\boldsymbol{k}_2] \cdot \dfrac{\boldsymbol{D}(\boldsymbol{r}'_i)}{n^2(\boldsymbol{r}'_i)}\}\boldsymbol{T}(\boldsymbol{r}'_i) \\ \boldsymbol{k}_4 &= -\{[\tilde{\boldsymbol{u}}(\boldsymbol{r}_i) + \Delta s\boldsymbol{k}_3] \cdot \dfrac{\boldsymbol{D}(\boldsymbol{r}_{i+1})}{n^2(\boldsymbol{r}_{i+1})}\}\boldsymbol{T}(\boldsymbol{r}_{i+1}) \\ \tilde{\boldsymbol{u}}_{i+1} &= \tilde{\boldsymbol{u}}_i + \frac{\Delta s}{6}(\boldsymbol{k}_1 + 2\boldsymbol{k}_2 + 2\boldsymbol{k}_3 + \boldsymbol{k}_4) \end{cases} \tag{5}$$

The calculation of $||\tilde{\boldsymbol{E}}(\boldsymbol{r})||$ is based on the intensity law of geometric field tracing [4].

$$||\tilde{\boldsymbol{E}}(\boldsymbol{r})|| = ||\tilde{\boldsymbol{E}}^{\text{in}}|| \cdot \sqrt{\frac{\hat{s}_z^{\text{in}} \cdot n^{\text{in}} \cdot \sigma^{\text{in}}}{\hat{s}_z(\boldsymbol{r}) \cdot n(\boldsymbol{r}) \cdot \sigma(\boldsymbol{r})}} \tag{6}$$

## Example

In this example, we are more interested in the effect of damping media. We input a Gaussian beam into a special damping media, whose refractive index contains a graded imaginary part. The parameters are as follows:

- Gaussian beam
  - Beam waist is 10 $\mu$m and we start from the Rayleigh length after beam waist.
  - Wavelength is 532 nm.
  - It is linear polarized. ($E_y = 0$)
- GRIN medium
  - The real part of refractive index is a constant, which is equal to that of air.
  - The imaginary part of refractive index is graded, as shown in Fig. 2
  - Length is 30 $\mu$m.

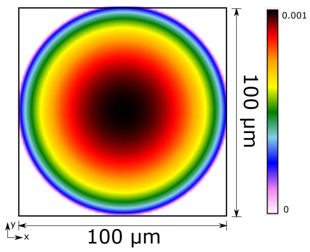


Figure 2: Im$\{\tilde{n}(\boldsymbol{r})\}$ in $x$

In this example, the incidence is an $E_x$-polarized field. As shown in Fig. 3(b) and (c), after the damping media, the amplitude of $E_x$ is reduced. At the same time, the output field contains a non-zero $E_y$-field . This crosstalk is caused by the graded damping part of media. Comparing the results from both the rigorous approach (FMM) and geometric field tracing, we see, although the amplitude of $E_y$ is quite small, geometric field tracing calculates it accurately. Therefore, this example also shows the validity of geometric field tracing in damping graded-index media.

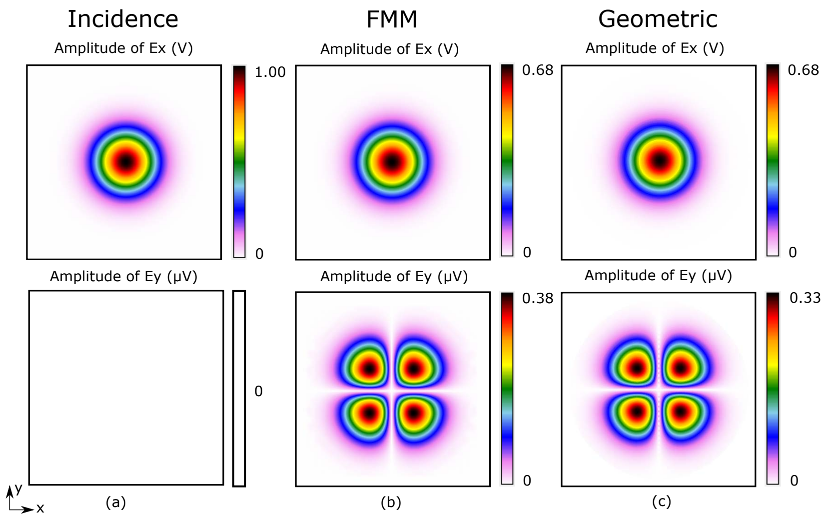


Figure 3: Amplitude of $E_x$ and $E_y$ of (a) incident field, (b) output field calculated by rigorous approach and (c) output field calculated by geometric field tracing.

## Conclusion

In this report, we propose the geometric field tracing in damping graded index media: the mathematical solutions of both equation of ray path and field in GRIN are given.

## Bibliography

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